

I am indebted to Dr. Morgan, of the Royal College of Science, for samples of various dyes.

NOTE.—Since the above was communicated to the Society Professor Langley, in a letter to *Nature*,* describes the great intensity of the inner corona, and how in consequence of this brilliance he was able to see *Mercury* in transit projected on the bright background, before it reached the Sun's limb. During the eclipse of 1878 he made a brief telescopic examination of the inner corona, and found it to be "a surprisingly definite filamentary structure . . . not disposed radially, or only so in the rudest sense, sharpest and much the brightest close to the disc, fading rapidly away into invisibility at a distance of 5' or more (possibly in some cases ten)."

This description of the inner corona agrees well, in respect to extension, brightness, and non-coincidence of the ordinary radial streamers, with the image of the corona seen in the light of the green coronal line.

Professor Langley further says "that while most interesting photographs of the inner coronal structure have recently been made, yet that this feature has not yet been done justice to even in the best of them I have seen, and that it perhaps cannot be, with our present means."

It would be interesting to see if by the use of a coloured screen this structural detail could be brought out more distinctly.

In a lecture delivered at the Society of Arts, 1900 March 12, on the "Photography of Colour," Mr. E. Sanger Shepherd states that he rejects liquid solutions and stained glass as unsuitable for colour filters, and pronounces in favour of aniline dyes sealed up in gelatine or collodin, between glasses; by experiment a number of these dyes have been found to be of fair permanency.

The Maximum Duration Possible for a Total Solar Eclipse.

By C. T. Whitmell, M.A., President of the Leeds Astronomical Society.

Total solar eclipses are among the grandest of natural phenomena. Prolonged totality is very uncommon, and its rarity invests it with exceptional interest.

The eclipse of Thales on 585 B.C. May 28, and that visible in Scotland on 1433 A.D. June 17, were remarkable for prolonged totality.

To come to more recent times, the eclipse of 1868 August 17 is said to have exceeded both those just referred to.

By the kindness of Dr. Downing an estimate of the duration of totality of this 1868 eclipse has been made, and—for local

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noon—the result is $6^m 49^s$. I have obtained a similar result. Governor Hennessy observed this eclipse from Barram Point, Borneo, where, in the afternoon, totality lasted $6^m 13^s$; and this is, I believe, the longest duration as yet actually observed.

Noon totality occurred in the Gulf of Siam, at E. longitude $102^\circ 38'$ and N. latitude $10^\circ 27'$. The conditions were very favourable, the Sun being not very far from apogee, the Moon near a favourable perigee, and almost in the zenith; whilst the observer, being in a low latitude, was situated on a rapidly moving part of the Earth.

The corresponding eclipse of the preceding *Saros* occurred on 1850 August 7. Mr. Paine (see *Monthly Notices, R.A.S.*, vol. xxix.) estimates the noon totality at $6^m 51^s$. It would be well to check this result, using the present eclipse values of the diameters of the Sun and Moon.

The eclipse of 1886 August 28 in the *Saros*, subsequent to 1868, had a totality of $6^m 35^s$, according to the *Nautical Almanac*. For the eclipse on 1901 May 17, the same authority gives a totality of $6^m 30^s$ for a place at which the occurrence of totality will be almost at local noon.

The two series of eclipses, 1850, 1868, 1886, &c., and 1883, 1901, 1919, &c., are amongst the most favourable in the whole history of astronomy.

Mr. Crommelin has kindly furnished me with the following results, calculated by him from Oppolzer's data:—

Date.	Duration at Noon.	Position of Noon Point.	
		Longitude.	Latitude.
1901 May 18	$6^m 41^s \cdot 6$	97° E.	2° S.
1919 May 29	$7^m 6^s \cdot 9$	18° W.	4° N.
1937 June 8	$7^m 19^s \cdot 9$	131° W.	10° N.
1955 June 20	$7^m 24^s \cdot 5$	117° E.	15° N.
1973 June 30	$7^m 19^s \cdot 6$	6° E.	19° N.
1991 July 11	$7^m 10^s \cdot 7$	105° W.	22° N.

The 1955 noon eclipse will occur not far from Manila, and its totality is the longest for many centuries.

Mr. Crommelin states that the above durations are trustworthy only to the nearest second, and that Oppolzer's data yield durations about eleven seconds in excess of those of the *Nautical Almanac*, which (I may add) are themselves probably some three seconds in excess of the results of observation.

I now proceed to consider the conditions necessary for the longest totality.

For simplicity, suppose at first that the Moon and the Sun move in the plane of the Earth's equator. When the centres of the three bodies are in a line, the Moon being between the Sun and the Earth, there will be a central solar eclipse at local noon. Suppose this to be total. Bring the Moon nearer to the Earth, and the size of her shadow, or umbra, will increase. Her

velocity also will increase. But the former factor far more than compensates for the latter, so that totality is lengthened by bringing the Moon nearer.

This may seem obvious, but I find that, if the Moon moved in a *circular* orbit, the increased velocity, when her distance was less than 104,000 miles, would more than compensate for the larger umbra, and the duration of totality would therefore fall off, if the Moon, moving in a circular orbit, had a radius vector of less than 104,000 miles.

Now take the Sun further away. The umbra again becomes larger, and the Sun's velocity diminishes. This latter result is a drawback, but is far more than compensated for by the increased size of the umbra. Thus, for a prolonged totality, the Moon should be as near, and the Sun as far off, as possible. The duration of totality increases with the size of the umbra, but diminishes with its velocity over the observer. Now, in the case supposed, the velocities of the Sun, the Moon, and the observer on the equator, are all in the same direction, *i.e.* from west to east, and, at local noon, are all perpendicular to the plane of the observer's meridian.

Thus the linear velocity of the umbra over the observer will be equal to that of the Moon, less that of the Sun in relation to the Moon, less that of the observer. We see, then, that so far as velocity is concerned it is better to have the Sun near and the Moon far away; but, as already explained, the increased size of the umbra (due to the nearness of the Moon and to the distance of the Sun) far more than compensates for its increased velocity.

Take the Earth's equatorial radius as 3963 miles, then, at mean distance, the linear sidereal velocity of the Moon is about 2288 miles an hour; but her linear synodical velocity in relation to the Sun is only 2117 miles per hour. The observer's velocity on the equator is 1037.5 miles per hour. Thus we get 1079.5 miles per hour as the noon velocity of the umbra over the observer. During the brief duration of even the longest totality the directions and values of these velocities will remain practically unchanged.

As a matter of fact, for mean distances of the Sun and Moon there would be no total eclipse, but the velocity values given above may be of interest.

There are really five conditions required for maximum totality.

1. The new Moon, at or very near a node, must be at the most favourable perigee possible.
2. The Sun must be at apogee.
3. During totality, which should be observed at local noon, the Moon's shadow must run along a parallel of latitude in order that the diurnal movement of the observer may be for the time parallel to the motion of the Moon, and thus produce its full effect in detaining him within the umbra.
4. The Sun and Moon must be in the zenith, so that the umbra may be as large as possible.

5. The observer must be on the equator, so that his velocity may be as great as possible.

The foregoing conditions could all be simultaneously fulfilled if the Sun and Moon moved in the plane of the equator; but, as this is not the case, we have to make a compromise consistent with the actual facts of nature.

We must keep conditions (1) and (2). Now it is a curious and fortunate thing that condition (3) can then also be fulfilled. The Moon's orbit makes an angle of about 5° with the ecliptic, and this is also about the value of the angle made by the descending ecliptic with that parallel of latitude which cuts it at its apogee point. The Moon must, of course, be in her ascending node, or very near it.

In other words, the Moon's declination, under the given conditions, will be at its monthly maximum, and so will scarcely vary during totality.

It is obvious, however, that conditions (4) and (5) cannot be satisfied simultaneously. Condition (4) puts the observer near the Tropic of Cancer. This increases the umbra, but lessens the observer's velocity. Condition (5) puts him on the equator. This increases his velocity, but diminishes the umbra, for the Moon is no longer in the zenith. We shall find that neither of these positions is the best possible, though (5) is more favourable than (4).

Suppose the Sun at apogee; what perigee distance can the Moon then have, consistently with her being new and at her node? For valuable help in determining this point I am indebted to Mr. Cowell, of Greenwich Observatory; but he is in no way responsible for my numerical results. Using Delaunay's "Lunar Theory," I compute that the Moon's horizontal parallax, under the above conditions, amounts to $61' 22''$. I shall be very glad to have this result confirmed. Under other conditions, such as the Moon being full, or 90° from her node, or the Sun being at perigee, the Moon's parallax may be greater, but these conditions are inconsistent with the problem under consideration.

Suppose the Earth to be a sphere, radius 3963 miles. Put the Sun at apogee, in N. declination $22^\circ 53' 55''$, with an angular semi-diameter of $15' 43''\cdot78$, this being calculated from the reduced mean distance semi-diameter, $15' 59''\cdot63$, used for eclipses. Let the new Moon be at perigee, and at her node, with a parallax of $61' 22''$. The linear semi-diameter of the Moon is about 1081.5 miles, and her geocentric angular semi-diameter is $16' 44''\cdot77$, if we use $15' 34''$ for the semi-diameter at mean distance, and not the smaller "eclipse" value.

Let the observer's geocentric latitude (ϕ) be $22^\circ 53' 55''$ N., and let the Moon be at her node and in the zenith at local noon. The observer's velocity will be less than at the equator, being only 955.74 miles an hour. The Moon's synodic velocity is 2310.50 miles per hour. The diameter of the umbra is 167.89 miles, and its velocity over the observer will be 1354.76 miles

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per hour. Putting T for the duration of totality in hours, we obtain for the duration at noon,

$$\text{N. latitude } \phi = 22^\circ 53' 55'', T = \frac{167.89}{1354.76} = 7^m 26^s.13 \quad . \quad A$$

In the case just considered the Moon was in the zenith. Now suppose her to have a small geocentric south latitude, so that her shadow falls on the equator. The umbra will be smaller, as the Moon is no longer in the zenith, but the observer's velocity will be greater, so that the velocity of the umbra in relation to the observer is diminished also :

$$\text{Latitude } \phi = 0^\circ 0' 0'', T = \frac{165.08}{1273.00} = 7^m 46^s.84 \quad . \quad B$$

distinctly exceeding (A).

Will this be the maximum totality? To determine this I formed an equation for T , and in this equation the latitude (ϕ) of the observer was practically the only independent variable. Differentiating this, and equating to zero in the usual way, T should be a maximum when $\phi = 4^\circ 47' 13''$ N. Using this value, we get

$$\text{N. latitude, } \phi = 4^\circ 47' 13'', T = \frac{166.12}{1276.62} = 7^m 48^s.45 \quad . \quad C$$

This exceeds (B) by $1^s.61$, and is really the maximum under the given conditions. T diminishes for latitudes either N. or S. of this value of ϕ . About N. latitude 9° or 10° T again becomes equal to its value for the equator.

We notice in (B), (C), (A) that the breadth of the umbra and its velocity are both increasing, but at different rates, and that their ratio becomes a maximum in case (C).

But the equation for the latitude has two roots, and the second value of ϕ is $192^\circ 32' 47''$. This means that we must put the observer in S. latitude $12^\circ 32' 47''$, and in longitude 180° away from his former position. We must also suppose the Earth transparent, so that the shadow can pass through to the side of it away from the Sun. Granting these suppositions, there will be a central total solar eclipse at local midnight.

$$\text{S. latitude, } \phi = 12^\circ 32' 47'', T = \frac{95.97}{3323.23} = 1^m 43^s.96 \quad . \quad D$$

This corresponds to *minimum* duration under the given conditions. The umbra, owing to the observer's increased distance, is much smaller, and its velocity is much greater, for now the observer's velocity has to be added to the synodical velocity of the Moon, for the observer is now moving in a direction opposite to that of the shadow.

This result is only of theoretical interest, but it well illustrates the generality of a mathematical formula, which disdains to take into account such a trifle as the Earth's opacity.

In an actual eclipse of the midnight Sun, the duration of totality is similarly shortened by the increased distance of the observer from the Moon, and by the condition that his velocity has to be added to that of the umbra.

To the foregoing results the following objections may be raised: that the Moon's radius has been taken too large, that the Earth's radius varies with the latitude, and that 3963·296 miles is a more accurate value of the equatorial radius. I have therefore re-calculated the results, using for the Moon's angular semi-diameter at mean distance the "eclipse" value $15' 32''\cdot65$, given in the *Nautical Almanac*. This corresponds to a radius of about 1080 miles, if we take 3963·296 miles for the Earth's equatorial radius, and reckon the Moon's horizontal parallax at mean distance to be $57' 2''\cdot70$. I have also allowed for the variation of the Earth's radius with the observer's latitude.

The durations are now as follows:—

$$\text{Equator, } \phi=0^{\circ} \ 0' \ 0'', T=\frac{161\cdot93}{1272\cdot99}=7^m 37^s\cdot94 \quad . \quad E$$

$$\text{N. latitude, } \phi=4^{\circ} \ 0' \ 0'', T=\frac{162\cdot82}{1275\cdot53}=7^m 39^s\cdot54 \quad . \quad F$$

$$\text{N. latitude, } \phi=4^{\circ} \ 51' \ 45'', T=\frac{162\cdot99}{1276\cdot75}=7^m 39^s\cdot58 \quad . \quad G$$

$$\text{N. latitude, } \phi=6^{\circ} \ 0' \ 0'', T=\frac{163\cdot20}{1278\cdot71}=7^m 39^s\cdot46 \quad . \quad H$$

The latitude for the maximum eclipse (G) is now a little ($4' 32''$) higher than before (C), and the duration is about 9 seconds less, a reduction due almost entirely to the Moon's smaller diameter of 2160 miles. But it will be noticed that the excess of duration of (G) over (E) is $1^s\cdot64$, about the same as it was in the case of (C) and (B). Of course, I assume not complete, but only relative, accuracy for the figures given. But I hope that, with the data used, there is no serious error in the results. For parallax $61' 22''$ the Moon's semi-diameter is $16' 43''\cdot30$.

The conclusion, then, is that, with the accepted present eclipse values of the diameters of the Sun and Moon, and with a lunar parallax of $61' 22''$, the maximum eclipse totality will occur near the beginning of July, at noon, in geocentric N. latitude about $4^{\circ} 52'$, and will last about $7^m 40^s$, the Sun being at apogee with a parallax of $8''\cdot70$. The Sun's and Moon's declinations are considered to be practically constant during totality.

If it were possible for the Sun and Moon to be in the plane of the equator, so that the Moon could be in the zenith, the eclipse would last $7^m 45^s\cdot88$, and the maximum would then be really on the equator. This, however, is impossible.

Mr. G. F. Chambers, in his comprehensive *Handbook of Astronomy* (vol. i. p. 268) quotes from Du Séjour a statement to the effect that maximum totality occurs on the equator, and lasts $7^m 58^s$. This statement has been copied into numerous text-

books. I have consulted the original paper in *Mémoires de l'Académie Royale des Sciences*, Paris, 1777, p. 318. Du Séjour states that the longest totality will occur at noon on the equator on July 2, with the Sun at apogee. The following data are used by him. Sun's semi-diameter, $15' 42''$; hourly motion in the ecliptic, $2' 23''$; declination, $22^{\circ} 50'$ N. The Moon is near her ascending node, in S. latitude $23' 57''$; hourly motion, reduced to the ecliptic, $38' 16''$; greatest polar parallax, $61' 17''$.

He does not give the Moon's semi-diameter, nor the relation between the polar and the equatorial radius of the Earth. With the ratio at present accepted, a polar parallax of $61' 17''$ would yield an equatorial one of $61' 29''$. He does not refer to the conditions which (I presume) were imposed upon the Moon's parallax by the lunar theory of his time, nor does he appear to be aware that the maximum totality is not actually on the equator. Taking Du Séjour's elements, and assuming for the Moon an equatorial parallax of $61' 29''$, and using her smaller semi-diameter, I get a result practically agreeing with his for an eclipse on the equator. But it seems to me that the parallax is larger than the lunar theory will permit under the conditions, and also that the semi-diameter of the Sun is too small.

Lord Grimthorpe, in his *Astronomy without Mathematics*, writes (p. 151) that "under a combination of the most favourable conditions totality may last something more than 7 minutes." But he bases his calculation upon an umbra only 148 miles in diameter, and this is decidedly under the mark. He also assumes the perigee distance of the Moon to be 221,600 miles, and this corresponds to a parallax of about $61' 29''$. If my computations are correct, this parallax, like that used by Du Séjour, exceeds the value allowed by the lunar theory under the given conditions. In addition to contributing interesting data as to future eclipses, Mr. Crommelin has been good enough to read over the MS. of this paper.

Leeds: 1900 February.

Observations of Saturn made at Juvisy Observatory in 1899.
By C. Flammarion.

Observations of *Saturn* were commenced here during the last apparition on 1899 June 1, and continued till July 30. The number of fine nights was abnormally great, a circumstance which more than compensated for the low altitude of the planet above the southern horizon.

The instrument employed was the Juvisy $10\frac{1}{4}$ -inch equatorial, bearing powers of 224, 308, 411 and 617, and the observations were made by M. Antoniadi and myself.